Rationalizing the denominator
Rational numbers

Another word for ‘fraction’ is ratio.

Any number that can be written as a fraction is a rational number.
Rational numbers

Some fractions convert to terminating decimals.

If you need to make them longer, you add zeros.

\[
\frac{1}{2} = .5 = .50 = .500 = .5000
\]

\[
\begin{array}{c}
2 \\
\hline
1.0
\end{array}
\]
Rational numbers

Some fractions convert to repeating decimals.

If you need to make them longer, you repeat the pattern.

\[
\begin{align*}
.33\ldots &= .333\ldots = .3333\ldots \\
\frac{1}{3} &= .33 \\
3\sqrt{1.00} &= .33
\end{align*}
\]
Irrational numbers

Any number that is not rational is irrational.

In English, ‘irrational’ means ‘not logical’, ‘crazy’.

Irrational numbers are easy to spot because they look wacky.
Irrational numbers

An irrational number is a decimal that never terminates and never repeats.

\[ \pi = 3.1415926... \]
Rationalizing the denominator

What if you have a fraction with an irrational number in the denominator?

Dividing it out is a royal pain!

\[ \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \]

\[ \begin{array}{c|c}
1.414 & 1.414 \\
\hline
1.000 & 1.000 \\
\hline
.7072 & .7072 \\
\hline
1414 & 1414 \\
\hline
1000.0000 & 1000.0000 \\
\end{array} \]
Rationalizing the denominator

How could you get rid of the irrational number in the denominator?

Multiply by something that would make it a whole number!

\[
\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}
\]
Rationalizing the denominator

Now let’s try that division!

WOW!

Is that ever easier!
Rationalizing the denominator

Let’s rationalize another denominator:

\[ \frac{1}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2}}{\sqrt[3]{8}} = \frac{\sqrt[3]{2}}{2} \]

Another way to think of that same problem –

You need **three** 2’s to break out from under a cube root sign.

\[ \frac{1}{\sqrt[3]{4}} = \frac{1}{\sqrt[3]{2 \cdot 2}} \]

\[ \frac{1}{\sqrt[3]{2 \cdot 2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2}}{2} \]
Rationalizing the denominator

Now for a really hard one that relies on multiplying by the conjugate.

Remember how to F.O.I.L.? 

\[
\frac{1}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}
\]
Rationalizing the denominator

Another hard one that relies on multiplying by the conjugate.

\[
\frac{1}{\sqrt{2} + \sqrt{3}} \cdot \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{\sqrt{2} - \sqrt{3}}{2 - 3} = \frac{\sqrt{2} - \sqrt{3}}{-1} = -\sqrt{2} + \sqrt{3}
\]

Remember how to F.O.I.L.?
Rationalizing the denominator

You are not allowed to leave radical signs in the denominator.

If it is a square root, multiply the top and bottom by the square root.

If it is a higher root, multiply by whatever it takes to break out of the radical sign.

If it is a binomial, multiply by the conjugate (same terms – opposite sign in the middle)