SIMPLIFYING RADICALS
“Radical” is another word for root:

Square root $\sqrt{9}$
cube root $\sqrt[3]{8}$
fourth root, etc. $\sqrt[4]{16}$
Definition of radicals

The **radical sign** is the “house” that the **radicand** lives in. \( \sqrt{} \)

The **index** tells you which kind of root it is.

\[ \sqrt[4]{16} \]

The radical sign with no index showing means the principal (or positive) square root.

\[ \sqrt{9} \]
Exponents and their corresponding roots are opposite operations – just like adding and subtracting or multiplying and dividing.

\[ \sqrt{9^2} = 9 \]

\[ (\sqrt{9})^2 = 9 \]

If you square a positive number and then take the square root of the answer, you’re right back where you started from.
Watch those signs!

Keep in mind that squaring a number produces a positive answer.

\[ \sqrt{(-9)^2} = \sqrt{81} = 9 \]

If you square a negative number and then take the square root of the answer, you’ve made it positive.
Fractional exponents

Another way to express a root, is to write a fractional exponent.

An exponent of $\frac{1}{2}$ means the square root
An exponent of $\frac{1}{3}$ means the cube root
An exponent of $\frac{1}{4}$ means the fourth root, etc
Fractional exponents

Remember that when you raise a power to a power you multiply the exponents.

\[
\sqrt{x^{16}} = (x^{16})^{\frac{1}{2}} = x^8
\]

\[
\sqrt{x^{21}} = (x^{21})^{\frac{1}{2}} = x^{10 \cdot \frac{1}{2}} = x^{10} \cdot x^{\frac{1}{2}} = x^{10} \sqrt{x}
\]
Alternate reality

When you are simplifying radicals, one way to do it is by thinking of the radical as a fractional exponent and applying the laws of exponents.

An alternate way to get the same answer, is by breaking the radicand down into prime factors and then using the index to tell you how many identical factors need to be in a group to move out of the house. For example:

\[ \sqrt{8x^3y^2} = \sqrt{2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y} = 2xy\sqrt{2x} \]

Since the index is understood to be 2, a pair of 2's can move out, a pair of x's can move out and a pair of y's can move out. Note, for each pair, only one shows on the outside.
Alternate reality - cube roots

When you are simplifying cube roots, the index is 3, therefore it takes a \textit{triple} of identical numbers or letters to move out of the house. For example:

\[
\sqrt[3]{16x^5y^6} = \sqrt[3]{2\cdot2\cdot2\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y\cdot y} 
\]

\[
= 2xyy\sqrt[3]{2x^2} = 2xy^2\cdot\sqrt[3]{2x^2} 
\]

Since the index is understood to be 3, a \textit{triple} of 2's can move out, a \textit{triple} of x's can move out and two \textit{triples} of y's can move out. Note, for each \textit{triple}, only one shows on the outside. The 2 y's on the outside are then multiplied together.
Some more examples

\[
\sqrt{49} = \sqrt{7 \cdot 7} = 7
\]

\[
\sqrt{24} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3} = 2\sqrt{6}
\]

\[
\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}
\]

If there is a negative outside the radical, the answer is negative.

\[-\sqrt{81} = -\sqrt{9 \cdot 9} = -9\]
Even roots of variables

If there is a letter inside the radical, and they haven’t specified that all variables represent positive numbers, then we need to use an absolute value symbol to force the variable to be a positive number.

\[ \sqrt{x^2} = x \] would not be true if \( x \) is a negative number

\[ \sqrt{x^2} = |x| \] guarantees that the answer is positive
Even roots of negative numbers

If there is a negative inside the radical, there is no solution within the set of real numbers.

\[ \sqrt{-64} \text{ NO solution} \]

That is because there are no two identical numbers (same signs) that multiply to give a negative.

Positive times positive = positive
negative times negative = positive
ALWAYS SIMPLIFY

Whenever you have a radical, simplify if you can!