Using the Discriminant to Determine the Nature of the Roots

What is the discriminant?

In general, the discriminant is an algebraic expression that gives information about the roots of a polynomial.

In the case of a quadratic equation the discriminant is the part of the quadratic equation that is under the radical sign.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What if the discriminant is positive?

If the discriminant is a positive number, then we get two real numbers as answers to the equation.

One of the answers comes from following the + path on the $\pm$

The other comes from following the − path.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 – 3x – 10 = 0$

$x^2 – 3x – 10 = 0$

a = 1

b = -3

c = -10

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{+3 \pm \sqrt{9 + 40}}{2}$$

$$x = \frac{+3 \pm \sqrt{49}}{2}$$

Discriminant = +49

$$x = \frac{+3 \pm 7}{2}$$

$$x = \frac{+3 + 7}{2} \text{ or } x = \frac{+3 - 7}{2}$$

$$x = \frac{10}{2} \quad \text{or} \quad x = \frac{-4}{2}$$

$$x = 5 \quad \text{or} \quad x = -2$$

$$x^2 = 5$$

$x^2 = 5$

$x^2 - 5 = 0$

$x^2 + 0x - 5 = 0$

$a = 1$

$b = 0$

$c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{0 + 20}}{2}$$

$$x = \frac{0 \pm \sqrt{20}}{2}$$

Discriminant = +20

$$x = \frac{+0 \pm \sqrt{4} * \sqrt{5}}{2}$$

$$x = \frac{+2\sqrt{5}}{2} \text{ or } x = \frac{-2\sqrt{5}}{2}$$

$$x = \sqrt{5} \text{ or } x = -\sqrt{5}$$

What if the discriminant is positive?

The two real numbers could be two rational numbers like -2, 5.

The two real numbers could be two irrational numbers like $-\sqrt{5}, +\sqrt{5}$

In either case, they are **real** roots of the equation because both rational numbers and irrational numbers are part of the set of real numbers.

What if the discriminant is zero?

If the discriminant equals zero then the entire radical drops out of the quadratic formula and you are left with one real number for the answer. The answer will be

$$\frac{-b \pm \sqrt{0}}{2a} = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$$

$$x^2 - 6x + 9 = 0$$

$x^2 - 6x + 9 = 0$

$a = 1$

$b = -6$

$c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 36}}{2}$$

$$x = \frac{6 \pm \sqrt{0}}{2}$$

Discriminant = 0

$$x = \frac{+6 \pm 0}{2}$$

$$x = \frac{+6 + 0}{2} \text{ or } x = \frac{6 - 0}{2}$$

$$x = 3 \text{ or } x = 3$$

What if the discriminant is negative?

If the discriminant is negative then you have the square root of a negative number which means that you have two imaginary answers.

One imaginary answer comes from following the path of the + and one comes from following the path of the – in the $\pm$

$$x^2 + 9 = 0$$

$$x^2 + 9 = 0$$

$$x^2 - 0x + 9 = 0$$

$a = 1$

$b = 0$

$c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{0 - 36}}{2}$$

$$x = \frac{0 \pm \sqrt{-36}}{2}$$

Discriminant = – 36

$$x = \frac{0 \pm 6i}{2}$$

$$x = \frac{+6i}{2} \text{ or } x = \frac{-6i}{2}$$

$$x = 3i \quad \text{or} \quad x = -3i$$

Shortcut!

If the question is just about what kind of answers to expect, or the "nature of the roots" you don't have to actually solve the equation and find the exact answers.

Just use the determinant to decide how many answers to expect (2 or 1) and whether they'll be real or imaginary.

The Nature of the Roots

Discriminant = $b^2 - 4ac$

Discriminant $+$ 2 real roots

Discriminant 0 1 real root

Discriminant $-$ 2 imaginary roots